

# F 14

## DEBYE EXPERIMENT

Christina Schwarz <sup>\*</sup>      Martin-I. Trappe <sup>†</sup>

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### Abstract

We investigate the behaviour of the specific heat of an  $\text{SiO}_2$ -crystal for the temperature range between 4.2 K and room temperature. The cooled  $\text{SiO}_2$ -crystal is heated by a low energy pulse whereas the temperature increase within the crystal is measured. Due to the experimental setup we also gain results for thermal and heat conductivity. Finally we verify the data theoretically given by the Debye model.

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<sup>\*</sup>Ch.Schwarz@urz.uni-hd.de

<sup>†</sup>Martin-Isbjoern.Trappe@gmx.de

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# I Introduction

Until the 19<sup>th</sup> century the heat capacity of solids was assumed to be constant for all temperatures. In the year 1819 Dulong and Petit were able to justify this law by a system of classical linear oscillators. Their model predicted the value of the heat capacity to be  $C_V = 3Nk_B$  for 1mol. Discrepancies of this behaviour were first explored by H. F. Weber in 1875 but could not be explained classically. Albert Einstein tried to give an explanation by assuming the solid to consist of  $3N$  independent quantum mechanical harmonic oscillators. In the cause of that he used simplifications that caused a wrong prediction in the limit of very low temperatures.

The model used nowadays was introduced by Peter Debye in 1911 interpreting a solid as a homogeneous isotropic elastic continuum with constant sound velocity. Here the thermal excitations are understood as elastic waves with longitudinal or transversal polarisation. The most important result of this theory was the correct description of the specific heat following the relation  $C \sim T^3$  for very low temperatures.

This work is mainly based on [2] and it is organized as follows: In section II we give a short introduction to the theory used for the evaluation of the measurements. Especially the connections between specific heat, thermal and heat conductivity are treated. We describe the experimental setup as well as the realization and finally present our results in section III. Concludingly we give a critical discussion in section IV.

## II Theory

### II. 1 Specific heat

The definition of the specific heat is given by  $C = T \frac{dS}{dT}$ . Evaluating the thermodynamical potentials  $H$  and  $S$  for constant  $p$  and  $V$  respectively one gains  $C_p$  and  $C_V$ :

For the enthalpy  $H$  holds

$$H = U + pV \text{ and } dH|_{p,N} = TdS. \quad (1)$$

So we obtain

$$C_p = T \frac{dS}{dT}|_{p,N} = \left( \frac{\partial H}{\partial T} \right)_p \quad (2)$$

Analogously, we derive  $C_V$  by exploiting the entropy  $S$ :

$$dS|_{V,N} = \frac{1}{T}dU \quad \implies \quad C_V = T \frac{dS}{dT}|_{V,N} = \left( \frac{\partial U}{\partial T} \right)_V \quad (3)$$

For  $C_p - C_V$  we can write

$$C_p - C_V = -T \left( \frac{\partial V}{\partial T} \right)_p^2 \left( \frac{\partial p}{\partial V} \right)_T = \frac{TV\beta^2}{\kappa_T} \quad (4)$$

using the isothermal compressibility  $\kappa_T = -\frac{1}{V} \left( \frac{\partial V}{\partial p} \right)_T$  and the coefficient of expansion  $\beta = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_p$ .

The approximation  $C_p \simeq C_V$  in solids holds for low temperatures, e.g. considering copper yields  $C_p - C_V \approx 10^{-4} \frac{\text{J}}{\text{Kmol}}$ .

## II. 2 Debye model

In our experiment we use the Debye model which provides correct descriptions in the cases of very low and very high temperatures respectively.

At first it presumes the solid to be a homogeneous isotropic elastic continuum. Therefore thermal excitations can be represented as superpositions of plane waves

$$\mathbf{U}(\mathbf{r}, t) = \mathbf{U}_0 e^{-i(\omega t - \mathbf{q} \cdot \mathbf{r})} \quad (5)$$

Of course, this approximation is only valid as long as the wavelength of this sound waves is sufficiently large compared to the lattice spacing  $a$  of the crystal.

As a second assumption one uses periodic boundary conditions because of the finite size of real crystals. This yields

$$q_x = \frac{2\pi}{L}n_x \quad q_y = \frac{2\pi}{L}n_y \quad q_z = \frac{2\pi}{L}n_z \quad (6)$$

for the components of  $\mathbf{q}$  while  $n_x, n_y, n_z \in \mathbb{Z}$ . Every triple  $n_x, n_y, n_z$  is connected to the absolute value of the wave vector by

$$q = \frac{2\pi}{L} \sqrt{n_x^2 + n_y^2 + n_z^2} \quad (7)$$

In addition the sound velocity  $v$  in the Debye model is assumed to be constant.

Considering one longitudinal ( $v_l$ ) and two transversal ( $v_t$ ) degenerate phonon branches one derives

$$Z_\omega = \frac{V\omega^3}{6\pi^2} \left( \frac{1}{v_l^3} + \frac{2}{v_t^3} \right) =: \frac{V}{2\pi^2} \frac{\omega^3}{v_D^3} \quad (8)$$

for the number of frequencies between 0 and  $\omega$ . Here  $v_D$  denotes the Debye velocity and  $V = L^3$  the volume of the crystal.

In the end one gains the density of states

$$D(\omega) = \frac{3}{2\pi^2} \frac{V}{v_D^3} \omega^2 \quad (9)$$

which is normalized by the number of degrees of freedom:

$$3N = \int_0^{\omega_D} D(\omega) d\omega \quad \Rightarrow \quad \omega_D = \frac{v_D}{a} (6\pi^2)^{\frac{1}{3}} \quad (10)$$

The density of states is used to derive the specific heat  $C_V$ . Therefore we calculate the intrinsic energy of the oscillations in the solid. A quantum mechanical derivation yields that the energy eigenvalues of our system are discrete and the energy difference between the energy eigenstates is given by  $\hbar\omega$  which is considered as the energy of one phonon. The Bose-Einstein-Distribution  $f(\hbar\omega, T)$  holds for phonons since they are spinless. Hence, for the Debye model one obtains the intrinsic energy

$$U(T) = \int_0^{\omega_D} \hbar\omega D(\omega) f(\hbar\omega, T) d\omega \quad (11)$$

Using the abbreviations  $x = \frac{\hbar\omega}{k_B T}$  and  $x_D = \frac{\hbar\omega_D}{k_B T}$  the specific heat is given by

$$C_V = \frac{\partial U}{\partial T} = 9Nk_B \left(\frac{T}{\theta}\right)^3 \int_0^{x_D} \frac{x^4 e^x}{(e^x - 1)^2} dx \quad (12)$$

while the Debye temperature  $\theta$  is defined by  $k_B\theta = \hbar\omega_D$ . We remark that the specific heat is entirely determined by the sound velocities  $v_t$  and  $v_l$ .

At this point we can consider two limits:

1. For  $T \rightarrow \infty$  the integral evaluates to  $\frac{1}{3} \left(\frac{\theta}{T}\right)^3$  and we end up in the law of Dulong and Petit

$$C_V = 3Nk_B \approx 25 \text{ Jmol}^{-1}\text{K}^{-1} \quad (13)$$

2. For  $T \rightarrow 0$  we get

$$C_V = \frac{12\pi^4}{5} Nk_B \left(\frac{T}{\theta}\right)^3 \quad (14)$$

Debye's approximation for low temperatures is valid in the temperature regime of  $T < 0.1\theta$  and the law of Dulong-Petit is achieved for  $\frac{T}{\theta} \approx 1$  (see [1]).

### II. 3 Heat conductivity

The coefficient  $\lambda$  of the heat conductivity for a constant temperature gradient is given by  $\mathbf{j} = -\lambda\nabla T$  where  $\mathbf{j}$  is the heat flux density. In general the heat conductivity as a

function of temperature depends on the predominating scattering processes within the solid. For instance, the heat transport in isolating crystals is essentially enforced by phonons. Especially, one introduces the so called approximation of dominating phonons which give the main contribution to the heat transport. Here only phonons with energy  $\hbar\omega \approx 2 - 3k_{\text{B}}T$  are considered.

From the kinetic gas theory then follows a proportionality between  $\lambda$  and the specific heat per mass unit:

$$\lambda = \frac{1}{3}C'vl \quad (15)$$

whereas  $v$  and  $l$  denote the velocity of the heat transporting particles and their mean free path. For the characteristics of  $\lambda$  one has to distinguish two processes:

1. The 3-phonon-scattering with conservation of quasi momentum (N-process) gives a contribution to the local thermal equilibrium but it is not connected to the heat conductivity.
2. In case of the U-process there is also a three phonon interaction but for the conservation of quasi momentum a reciprocal lattice vector is needed. This process causes an exponential increase of the mean free path while the temperature is decreasing. According to (15)  $\lambda$  is proportional to  $l(T) \propto e^{\frac{\theta}{2T}}$  since  $C_V = 3Nk_{\text{B}}$  and  $v \approx \text{const}$  for high temperatures. As the mean free path reaches the length of the sample the heat conductivity cannot raise any longer and decreases via  $\lambda = \frac{1}{3}C'_Vvd \propto T^3$ .

## II. 4 Temperature distribution in the sample

Our problem of heat conductivity is totally determined by the differential equation of heat conductivity

$$\frac{\partial^2 T(x, t)}{\partial x^2} = \frac{1}{k^2} \frac{\partial T(x, t)}{\partial t} \quad (16)$$

and the coefficient of thermal conductivity

$$k = \sqrt{\frac{\lambda}{C'\rho}} \quad (17)$$

and the following start and boundary conditions:

- (i)  $T(x, t_0 = 0) = \frac{Q_{\text{tot}}}{\rho FC'} \delta(x) + T_0 =: D \delta(x) + T_0$
- (ii)  $T(L, t) = T_0$

Thereby the sample is regarded as a homogeneous cylinder with length  $L$  and cross sectional area  $F$ . The whole energy absorbed by the sample is denoted as  $Q_{\text{tot}}$ . Furthermore the parasitic heat loss is neglected.

The solution of equation (16) is brought about Laplace transformation and in the end one derives a series as representation of the solution. The evaluation of this series being very complicated in most cases we perform an approximation that restricts the series to the first term and one obtains the variation in temperature:

$$\xi(x, t) = T(x, t) - T_0 = \frac{D}{k\sqrt{\pi t}} e^{-\frac{x^2}{4k^2t}} \quad (18)$$

At this the occurring error does not exceed 1%.

The maximum of  $\xi$  is evaluated to

$$\xi_{\text{max}} = \frac{D}{k\sqrt{\pi t_{\text{max}}e}} = \frac{Q_{\text{tot}}\sqrt{2}}{\rho F C' x_p \sqrt{\pi e}} \quad , \quad \text{at time } t_{\text{max}} = \frac{x^2}{2k^2} \quad (19)$$

The experimental setup restricts us to measure the variation in temperature at  $x = x_p$ . Thereby the optimal procedure for the evaluation is to plot

$$\ln[\xi(x_p, t)\sqrt{t}] = -\frac{x_p^2}{4k^2t} + \ln\left(\frac{D}{k\sqrt{\pi}}\right) \quad (20)$$

The slope of this graph provides  $k^2$  and as an implication  $t_{\text{max}}$  which is more precise than the directly measured value of  $t_{\text{max}}$ . Then  $\xi_{\text{max}}$  – as necessary quantity for  $C'_V$  – can be calculated. From equation (19) the specific heat is immediately derived as

$$C'_V = \frac{1}{\rho F x_p \sqrt{\pi e/2}} \frac{Q_{\text{tot}}}{\xi_{\text{max}}} \quad (21)$$

### III Realization

First we give a brief sketch of our experimental setup. We fix one side of the sample to the temperature  $T_0$  while the other side is heated by a short pulse. Then the time dependent variation of the temperature is recorded by a storage oscilloscope. By degrees the temperature in the whole crystal raises and considering an arbitrary point within the crystal a temperature maximum will pass through after a resolute time. Finally the temperature will approach the surrounding temperature  $T_0$ . For us looking at a fixed point in the crystal the temporal difference between the pulse and the achievement of the maximum yields the coefficient of heat conductivity while the magnitude of the increase of temperature gives the specific heat.

### III. 1 Setup

#### III.1.1 Cryostat

The main piece of the apparatus is a helium cryostat as shown in Figure 1. The outer shell consists of two dewars separated by an isolating vacuum. The outer dewar will be filled with liquid  $N_2$  in order to precool the inner one as well as to insulate the latter from thermal radiation. The inner dewar will be filled with liquid  $^4He$ . Here the sample is stuck into a separate vacuum chamber.

The temperature range of such a cryostat covers temperatures from 4.2K to 300K but its handling requires a sophisticated pump system (see Figure 2).

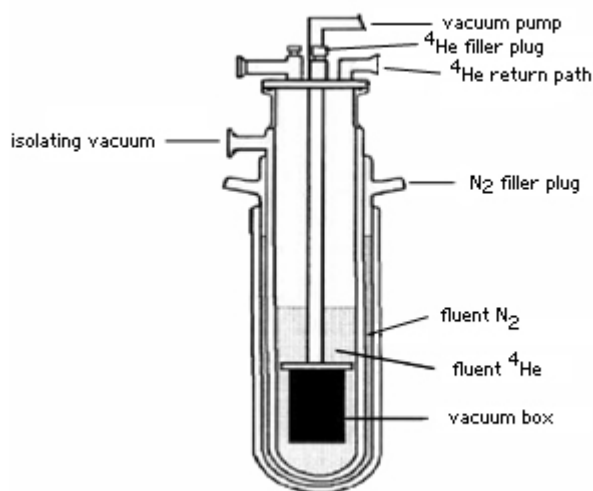


Figure 1: Schematic diagram of a helium cryostat

#### III.1.2 sample

As already said above we use a cylindrical  $SiO_2$ -monocrystal consisting of two pieces with diameters of 10mm and lengths of 7mm and 14mm respectively (see Figure 3). This crystal is plugged in a block of copper that serves as the heat reservoir with temperature  $T_0$ . The density of quartz constitutes  $2.65 \frac{g}{cm^3}$ . Between the bottom and the upper part three thermal elements connected in series are embedded. The heater is installed onto the top of the crystal while the bottom is connected to the heat reservoir with constant temperature  $T_0$ .

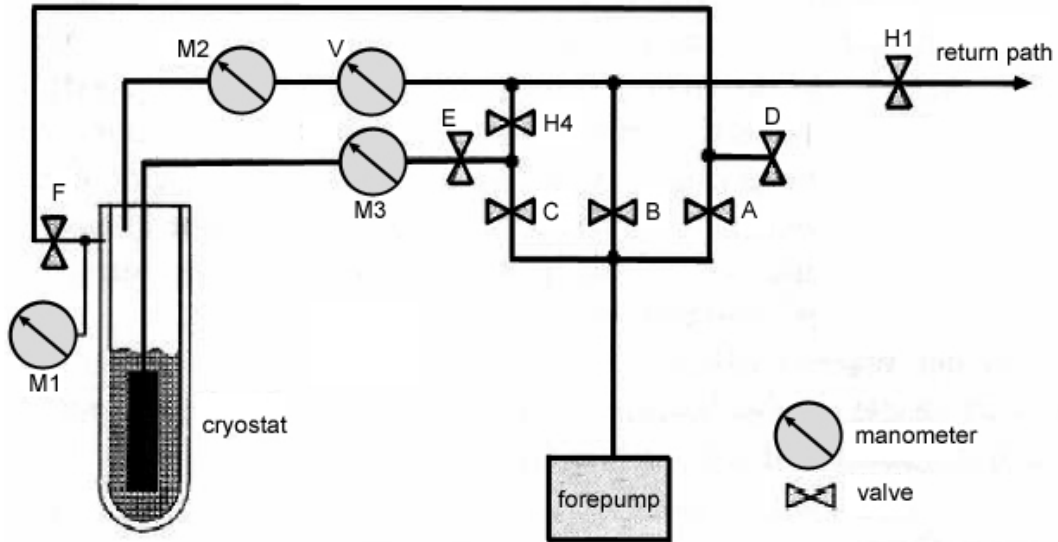


Figure 2: Schematic diagram of the gas and pump system

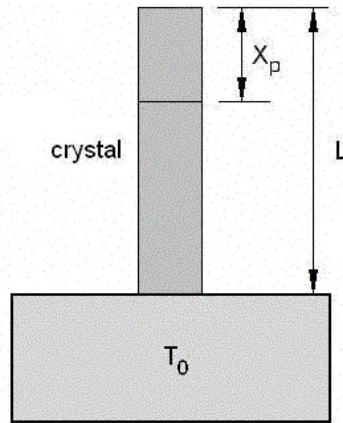


Figure 3: Configuration of the crystal

### III.1.3 Electronic measuring devices

Mainly the electronic devices in this experiment are used for the generation of heat pulses and the recording of the resulting temperature variation. These devices are shown in the schematic diagram (see Figure 4) and the PID controller adjusting  $T_0$ .

To measure the temperature difference between the position  $x_p$  in the crystal and the heat reservoir the above mentioned thermal couples are used. The appropriate one for our temperature range is an Au(0.03% Fe) thermal couple. The calibration between the displayed voltage and the temperature difference is depending on the temperature of the surrounding environment as shown in appendix A. The resulting voltage is in the magnitude of  $\mu V$  and therefore is amplified by a factor of 10000 per thermal couple.

Since one wants to know the absorbed energy from the heat pulse the resistance of the

heater has to be determined for the purpose of which we utilize a separate circuit supplied by a constant current of  $100\mu\text{A}$ .

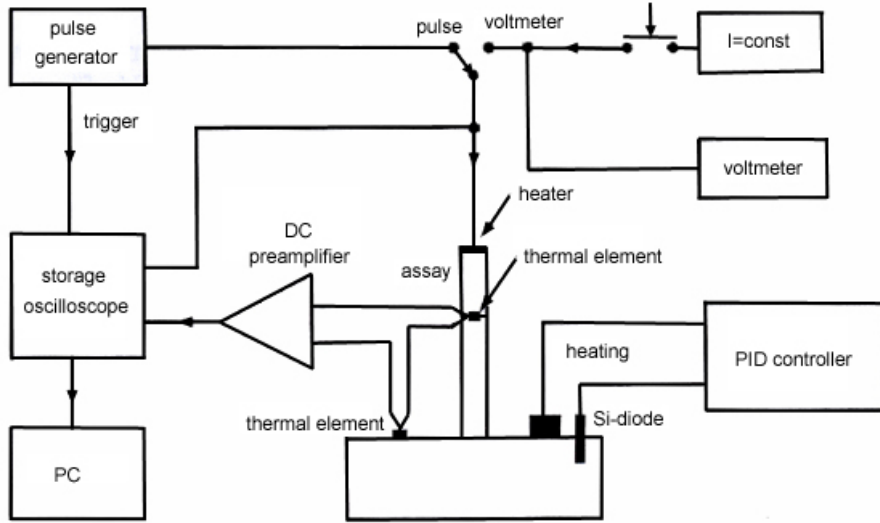


Figure 4: Block diagram of the electronics

## III. 2 Measurement

### III.2.1 Prearrangement

Before starting the measurement the cryostat has to be cooled down first by liquid  $\text{N}_2$  (77K) and then by liquid  $^4\text{He}$  (4.2K). This procedure is realized in several steps which are briefly discussed in the following. For a more detailed consideration of this comprehensible pump process the reader is referred to Figure 2.

The vacuum chamber of the sample is exhausted with a final pressure of  $p < 3 \cdot 10^{-2}\text{mbar}$ . This is needed for negligible heat loss during the measurement.

The next task is the preparation of an isolating vacuum between the  $\text{N}_2$ -dewar and the  $^4\text{He}$ -dewar. Due to an effective cooling with nitrogen the exchange gas must have a pressure in the range of 1mbar. After filling in liquid helium the remaining air will freeze out and the  $^4\text{He}$ -dewar is thermally isolated.

But first the  $^4\text{He}$ -dewar is filled with gaseous  $^4\text{He}$  from the return line (see Figure 2) in order to supply contact gas for precooling the sample to 77K. Now the liquid nitrogen is filled in. After the cryostat is cooled down the liquid helium is filled in the helium dewar and the sample finally reaches the aimed temperature of 4.2K.

For an elaborate presentation of the measuring process refer to section III.2.2.

During this prearrangement one still has to consider several aspects:

- In order to achieve a constant pressure during the several measurements one has to exhaust the vacuum chamber since all surfaces within the vacuum chamber emit atoms and the whole setup contains glue which always evaporates. In addition there is assumed to be a leak in the apparatus also increasing the pressure.
- Since helium evaporates permanently the helium return line has always to be opened to avoid oversized pressure in the pipes.
- Reaching higher temperature ranges ( $\approx 70\text{K}$ ) a higher pressure in the vacuum chamber of about  $10^{-2}\text{mbar}$  is required for a sufficiently fast temperature adjustment.

### III.2.2 Measuring process

Our aim is to measure the temperature increase at the point  $x_p$  in the sample as a function of time at about thirty different temperatures between 4.2K and 300K. To perform a convenient evaluation later on we choose exponentially increased heat reservoir temperatures in order to represent the data equidistantly in a logarithmic scale. These temperatures are regulated by a PID-controller which provides adequate heating outputs for the different temperature ranges.

The crystal thermalizes during the heating process induced by the PID-controller. Hereby the variation in temperature is additionally superimposed by the obligatory noise and can be recorded by the storage oscilloscope. If the magnitudes of this variation and the noise are comparable the temperature of the whole crystal is assumed to be constant. Once the heat reservoir temperature is adjusted the measurement can be started.

The pulse generator induces a pulse that is passed to heater on top of the sample. The length and height of the pulse can be preset. Of course, this feature is necessary since a high energy input at low temperatures leads to an enormous relative variation in temperature. So the temperature of the heat reservoir cannot be assumed to be fixed and the error of the temperature increase would be enlarged out of scale. It is important that the length of the pulse is short compared to the dimension of the measured time to the temperature maximum since the theory demands the shape of the pulse to be a delta distribution. For recording the pulse at the oscilloscope the pulse generator sends a trigger signal when the pulse is started.

The storage oscilloscope registers the incoming signal of the thermal couples and the pulse during a time period of about 1s. Then this data is transmitted to a PC for further processing.

To derive the specific heat we have to determine the energy deposited by the heater. Therefore its resistance that varies with temperature has to be measured in a separate circuit that is supplied by a constant current of  $100\mu\text{A}$  (see Figure 4).

### III. 3 Results and Discussion

A table of all measured and derived quantities is presented in appendix B.

#### III.3.1 Specific heat

The evaluation of the recorded data contains the maximal temperature increase  $\xi_{\max}$  and the elapsed time  $t_{\max}$  to this maximum at every considered temperature. In addition the deposited energy in the sample is calculated for every measurement by

$$Q_{\text{tot}} = \frac{U_{\text{pulse}}^2}{R_{\text{heater}}} t_{\text{pulse}} \quad (22)$$

Using equation (21) and the measured quantities  $\xi_{\max}$  and  $Q_{\text{tot}}$  one gains the specific heat per mass unit  $C'_V$ . Performing this procedure for every regarded temperature we plot  $C'_V$  as a function of temperature  $T$  and fit our data with  $f(T) = a \cdot T^3$  in the range from 8K to 40K obtaining  $a = 1.153(14) \cdot 10^{-4}$  (see Figure 5).

In general  $C'_V$  and  $C_V$  are related via  $C'_V \cdot c = C_V$  with  $c = M_{\frac{1}{3}\text{mol}}(\text{SiO}_2) = 0.02 \frac{\text{kg}}{\text{mol}}$  in our case. The Debye temperature of  $\text{SiO}_2$  is then given by  $\theta = \sqrt[3]{\frac{12\pi^4}{5ac} Nk_B}$  and evaluates to 945K with an error of  $\Delta\theta = \frac{1}{3}\theta \frac{\Delta a}{a} \approx 4\text{K}$ . Here we assumed to have 1mol harmonic oscillators for  $\frac{1}{3}\text{mol SiO}_2$ .

The bibliographic reference to the Debye temperature of crystal quartz (hexagonal lattice structure) is given by  $\theta_{\text{lit}} = 470\text{K}$  [3]. For the explanation of this disagreement we now discuss our derivation of  $\theta$  from Figure 5.

In our experiment the validity of the Debye model is restricted by the splitting of the quartz monocrystal into two parts which are connected via an adhesive film of Stycast. So the assumption of a monocrystal obviously leads to a deviation between theory and our experimental setup.

Another influence is the fitting procedure we performed for the temperature range from 8K to 40K.

Especially, our results for temperatures below 8K are not reliable. One reason is the big error of the specific heat which occurs from the relatively high energy input of the heater related to the low surrounding temperature. For instance, the maximal temperature increase at the reading point  $T = 4.2\text{K}$  is  $\xi_{\max} \approx 1\text{K}$  (see appendix B), i.e. we have

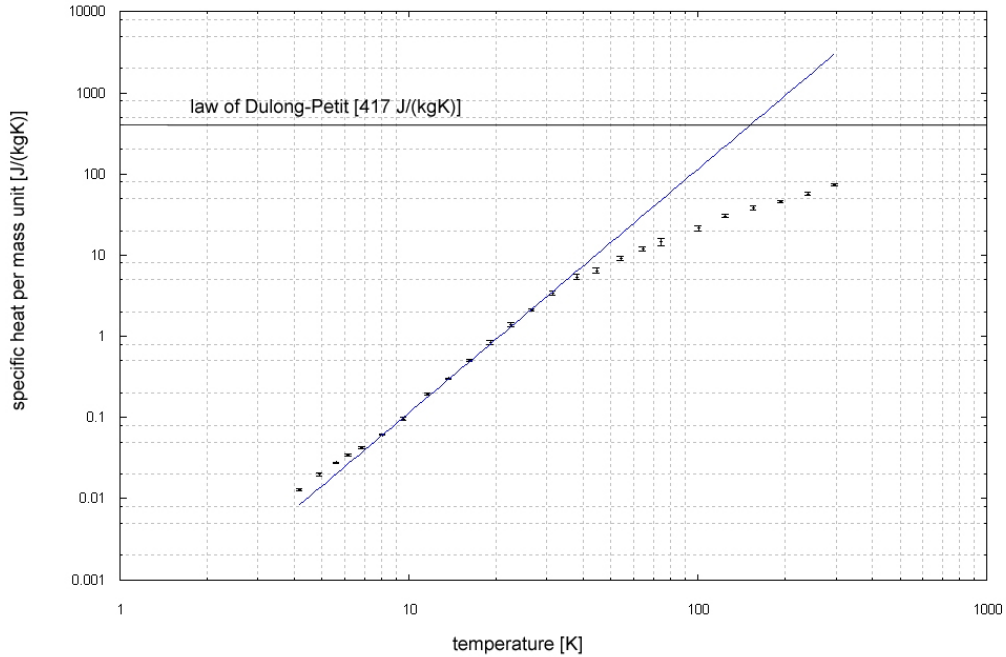


Figure 5: Specific heat per mass unit  $C'_V(T)$

an additional relative error in  $\xi_{\max}$  of about  $(\Delta\xi_{\max})_{\text{add}} \approx 10\%$  which is comprehensible considering appendix A. Because of  $C'_V \propto \frac{1}{\xi_{\max}}$  we also obtain an additional relative error of 10% for  $T < 8\text{K}$ . For an improved evaluation one could derive a mean temperature and choose it as the temperature of the heat reservoir. However, for the fit function we use the measuring results for  $T > 8\text{K}$  providing more precise data.

We also cut the fit range at 40K since one would expect a  $T^3$ -dependence for  $C'_V$  only at temperatures  $T < 0.1\theta$ .

Furthermore, we only considered temperatures up to  $T = 300\text{K}$  which gives

$$\frac{T}{\theta} = \frac{300\text{K}}{470\text{K}} \approx 0.64$$

Using the Debye model the law of Dulong-Petit is approximately achieved for temperatures  $T > \theta$  as mentioned in section II. 2. Thus we only can state that the curve levels for higher temperatures as we can see in Figure 5 and so we assume the law of Dulong-Petit to be valid.

### III.3.2 Thermal conductivity

We choose two methods to derive the coefficient of thermal conductivity  $k$ . At first one can directly determine  $k$  from equation (19) as  $k = \frac{x_p}{\sqrt{2t_{\max}}}$ . We obtain  $t_{\max}$  from the time elapsed of the temperature increase but for the maximum being not very sharp in

general one should prefer the second method which was described in section II. 4. Here  $k$  is calculated from the slope of the graph provided by equation (20). For every considered temperature we have to plot  $\ln[\xi(x_p, t)\sqrt{t}]$  as a function of  $\frac{1}{t}$  and fit a straight line with slope  $m$  in the region of the raising temperature increase. Because we measured the voltage  $U$  from the thermal couple one has to employ the following consideration. The relation between  $\xi$  and  $U$  is given by  $\xi = \text{const} \cdot U$  and because of  $\ln(\xi\sqrt{t}) = \ln(U\sqrt{t}) + \text{const}$  one obtains the same slopes  $\frac{x_p^2}{4k^2}$ .

We must not consider the range after the temperature maximum because here the approximation of  $\xi(x_p, t)$  (see equation (18)) is insufficient. We assume the data for very small times not to be reliable because of the broadened heat pulse which in theory is a  $\delta$ -function. As seen exemplarily in the Figures 6 and 7 the data in this time regime does not coincide with the fit line. But the main data of the temperature increase can be fit excellently by a line.

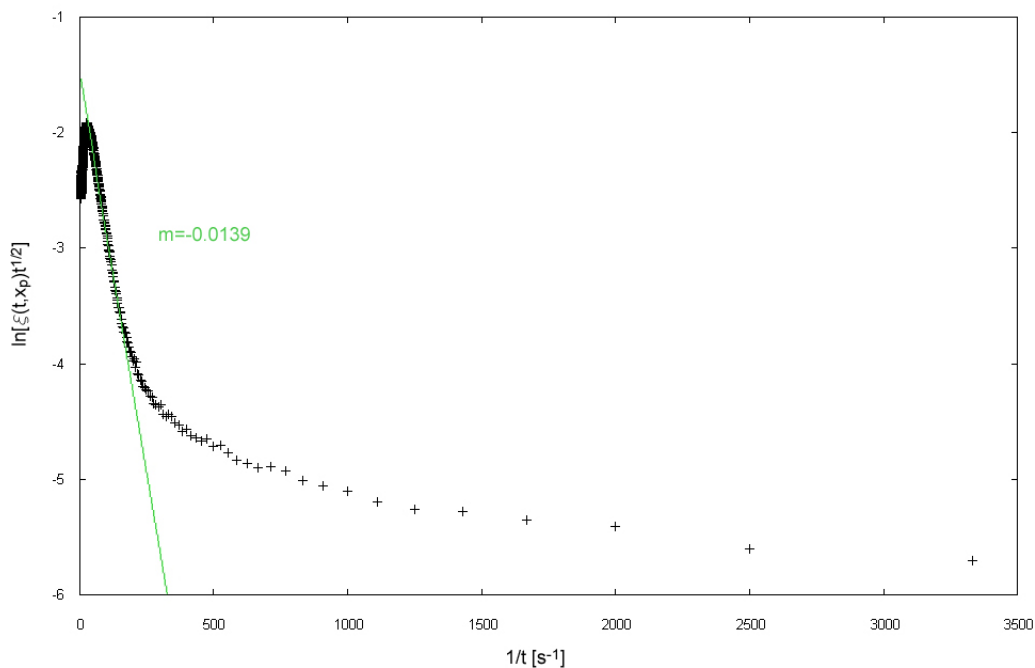


Figure 6: Time elapsed according to equation (20) for 5.6K

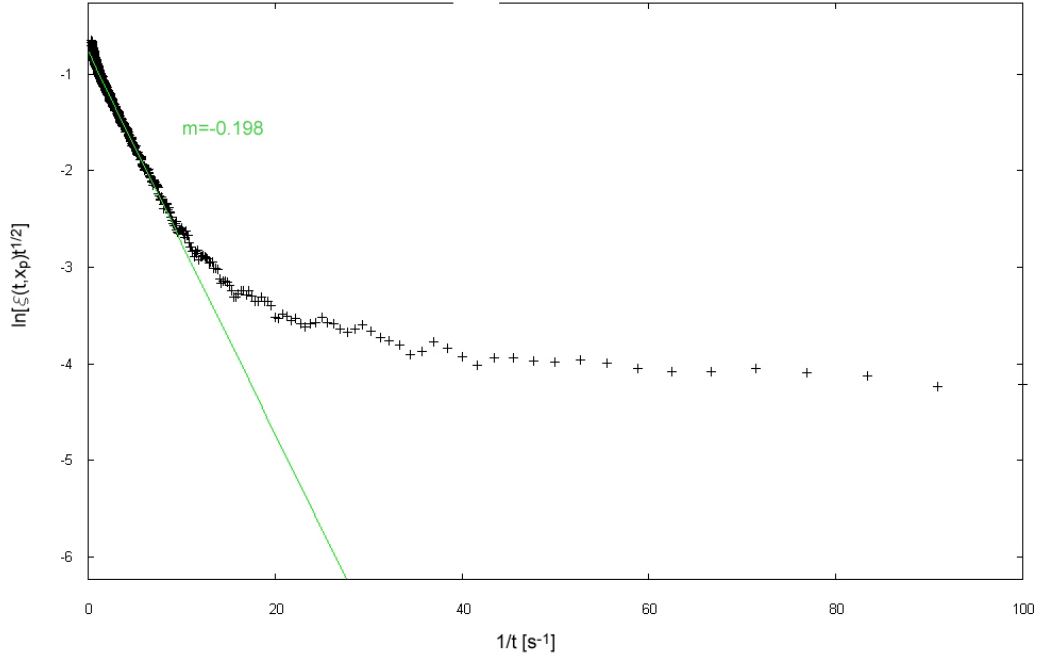


Figure 7: Time elapsed according to equation (20) for 64.3K

The fitting procedure is performed for every regarded temperature in the end giving  $k = \frac{x_p}{2\sqrt{-m}}$  as a function of temperature.

One recognizes a satisfying correspondence between both methods illustrated in Figure 8. In our case the second method should be preferred for further calculations because of its smaller errors compared to those of  $k$  obtained by  $k = \frac{x_p}{\sqrt{2t_{\max}}}$ .

Having measured  $C'_V(T)$  and  $k(T)$  we can derive the heat conductivity  $\lambda(T)$  using  $\lambda = k^2 C'_V \rho$  (see equation (17)). Since the behaviour of heat conductivity with respect to temperature is available from literature (see Figure 10) we are able to compare our measured result of the thermal conductivity with the predicted one. Therefore we investigate  $\lambda$  as a function of  $T$  in the following section.

For completeness we describe the general theoretical characteristics of  $k(T)$ . The following relations qualitatively determine the graph of  $k(T)$ :

- for low temperatures

$$\left. \begin{array}{l} \lambda \propto T^3 \\ C'_V \propto T^3 \end{array} \right\} \implies k = \text{const} \quad (23)$$

- for high temperatures

$$\left. \begin{array}{l} \lambda \propto \exp\left(\frac{\theta}{2T}\right) \\ C'_V = \text{const} \end{array} \right\} \implies k \propto \exp\left(\frac{\theta}{4T}\right) \quad (24)$$

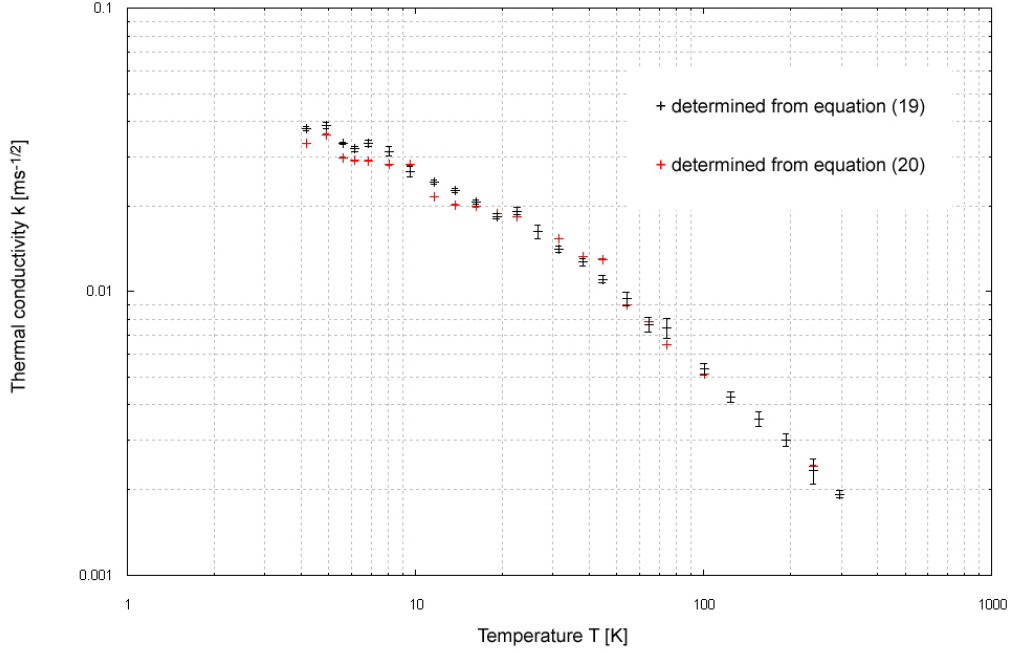


Figure 8: Comparison of the thermal conductivity derived from equation (19) and (20) respectively

### III.3.3 Heat conductivity

Having determined  $k$  and  $C'_V$  equation (17) yields the coefficient of heat conductivity (see Figure 9). For  $\lambda$  we expect a temperature dependence  $\lambda \propto T^3$  for low temperatures and  $\lambda \propto e^{\frac{\theta}{2T}}$  for high temperatures as already mentioned in section II. 3 (for illustration see Figure 10 where  $\lambda(T)$  of NaF is shown). The fit in the range between 40K and 250K yields the fit function  $g(T) = 0.77(7) \cdot \exp(\frac{57(7)}{T})$  which implies  $\theta = 114(14)$ K. Thus it is not possible to verify the prediction with respect to high temperatures since the errors of our small amount of data are too large for an appropriate fit.

For low temperatures we find a  $\lambda \propto T^2$ -dependence according to the fit function  $f(T) = 0.0016(3) \cdot T^{2.07(7)}$  which is characteristic for amorphous materials. So we assume that this discrepancy is based on the altered structure of the sample. For the measurement the needed monocrystal is divided into two pieces connected via an amorphous layer that affects the heat conductivity.

As already said in section III.3.2 we are now able to compare the behaviour of thermal conductivity with theory. But since we obtained a large deviation of  $\lambda(T)$  with respect to the expected values the deviation of the thermal conductivity  $k$  from the expected values achieves a similar magnitude.

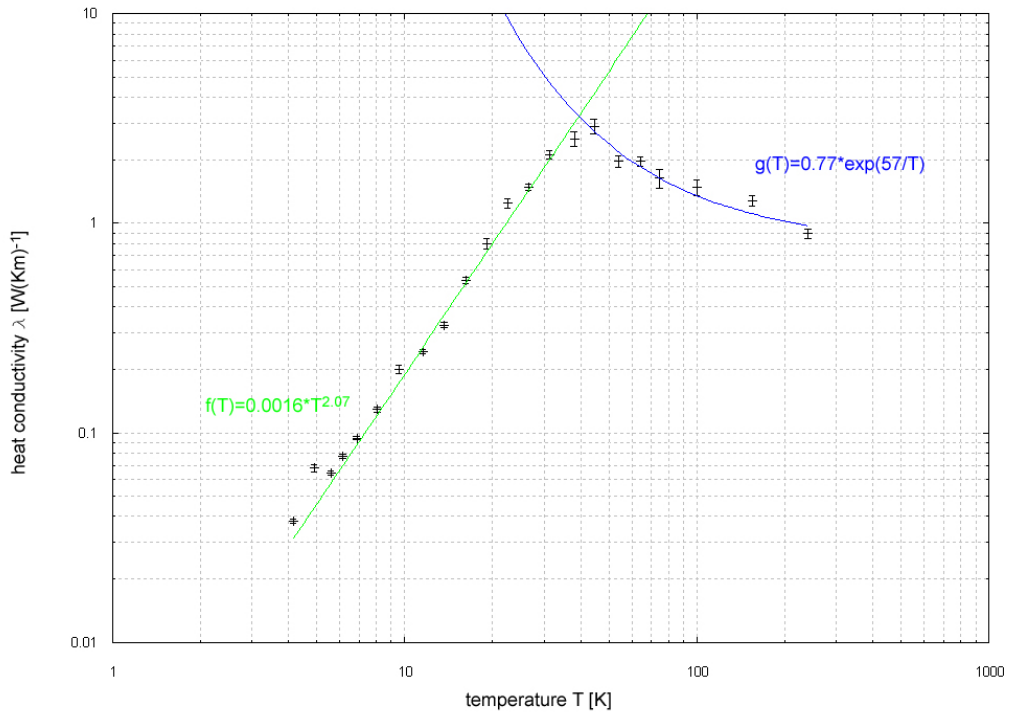


Figure 9: Heat conductivity according to equation (17)

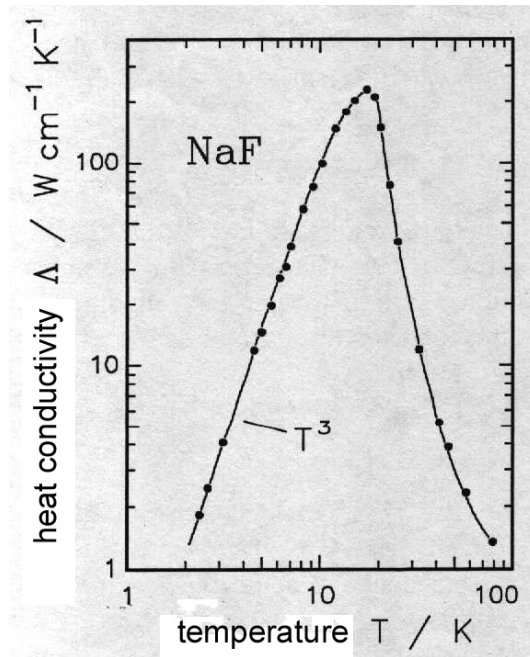


Figure 10: Heat conductivity of NaF.

## IV Conclusions

In this experiment we considered the specific heat of an  $\text{SiO}_2$ -crystal for the temperature range between 4.2 K and room temperature. The lowest temperatures are achieved by using a cryostat cooled with helium and nitrogen. We measured the time elapsed of the temperature increase within the crystal induced by a heat pulse. The experimental setup yields the specific heat as a function of  $T$ , the thermal as well as the heat conductivity and additionally the Debye temperature. For a more precise consideration of the law of Dulong-Petit one has to take in account temperatures above  $\theta = 470\text{K}$ .

Furthermore the Debye temperature as an indicator of the validity of  $C_V = \frac{12\pi^4}{5} N k_B \left(\frac{T}{\theta}\right)^3$  has been determined to  $\theta_{\text{exp}} = 945(4)\text{K}$  but we could not confirm the bibliographical reference of  $\theta_{\text{lit}} = 470\text{K}$ . The very small relative error of  $\theta_{\text{exp}}$  implies a correct measurement but a wrong ansatz. We assume the most likely source of errors to be the divided crystal. A mathematical consideration yields that the thermal conductivity  $k$  can be derived from the time between the heat pulse and the maximal temperature increase. and  $C'_V$  we obtained the heat conductivity  $\lambda$  with a  $T^2$ -dependence instead of the expected  $T^3$ -law for low temperatures. The unsatisfactory data in the region of high temperatures yields the Debye temperature  $\theta = 114(14)\text{K}$  assuming an exponential decay. Because of the relation between  $\lambda$  and  $k$  via the specific heat  $C_V$  we also cannot give satisfactory quantitative conclusions for  $k$  but we revealed the qualitative behaviour of  $k(T)$  as expected from theory.

# A Transformation between thermal voltage and temperature variation

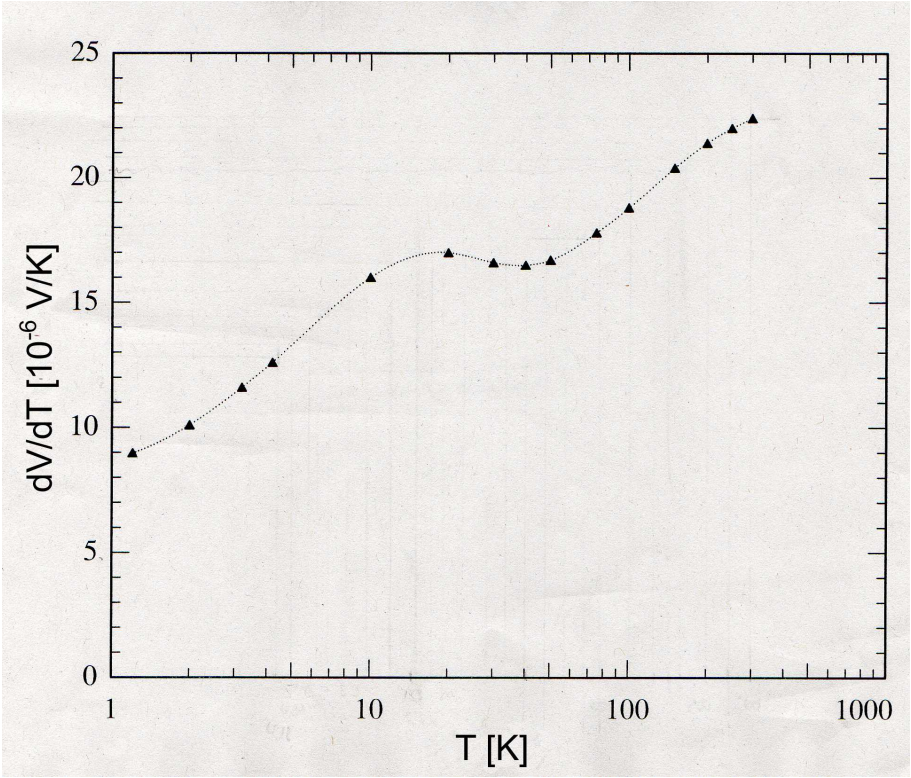


Figure 11: Transformation 'voltage  $\leftrightarrow$  temperature' of the used thermal element

## B Tables of measurements and derived quantities

$T$ [K]	$t_{\max}$ [ms]	$\Delta t_{\max}$ [ms]	$U_{\max}$ [mV]	$\Delta U_{\max}$ [mV]	$\frac{dU}{dT}$ [ $\frac{V}{K}$ ]	$\Delta \frac{dU}{dT}$ [ $\frac{V}{K}$ ]	$U_{\text{pulse}}$ [V]	$t_{\text{pulse}}$ [ms]	$U_{\text{heater}}$ [mV]
4.18	17.2	0.6	376	6	12.8	0.2	1.01	3.56	9.61
4.91	16.4	0.8	272	10	13.3	0.2	1.05	3.56	9.61
5.6	22	0.4	672	12	14	0.2	1.58	5.12	9.61
6.16	24	1	552	8	14.4	0.2	1.58	5.12	9.61
6.87	22	1	460	6	14.7	0.2	1.58	5.12	9.61
8.09	25	2	330	7	15.4	0.2	1.58	5.1	9.61
9.58	35	3	214	9	16	0.2	1.56	5.08	9.61
11.59	41.2	1.6	432	10	16.6	0.2	1.96	12.8	9.61
13.71	47	1.5	416	10	16.8	0.2	2.36	12.9	9.62
16.2	57	2	348	10	16.9	0.2	2.8	12.8	9.62
19.12	72	3	208	11	17	0.2	2.8	12.8	9.7
22.55	66	4	240	12	16.9	0.2	3.88	12.8	9.72
26.64	92	10	340	10	16.6	0.2	5.76	12.8	9.75
31.38	122	6	290	13	16.5	0.2	6.76	12.9	9.81
38.1	152	8	190	15	16.5	0.2	6.88	13	9.9
44.54	200	12	148	12	16.6	0.2	6.72	12.8	10
53.93	276	30	182	11	17.1	0.2	8.8	12.8	10.11
64.3	420	50	308	16	17.6	0.2	13	12.8	10.23
74.7	440	70	248	25	18.1	0.2	12.8	12.8	10.38
100.2	860	80	150	13	19	0.2	15.2	7.8	10.63
124.47	1360	120	284	9	19.8	0.2	18	14.8	10.93
155.18	1940	220	230	14	20.8	0.2	18	14.8	11.28
193.32	2720	280	388	14	21.4	0.2	25.6	14.8	11.72
240.8	4500	900	304	16	22	0.2	25.6	14.8	12.26
297.2	6600	400	500	8	22.5	0.2	21	48.4	13.02

Table I:  $\frac{dU}{dT}$  is read off Figure 11.  $\Delta$  denotes the error of the corresponding quantity.

$\xi_{\max}$ [K]	$\Delta \xi_{\max}$ [K]	$R_{\text{heater}}$ [ $\Omega$ ]	$Q_{\text{tot}}$ [J]	$C'_V$ [ $\frac{J}{\text{kg K}}$ ]	$\Delta C'_V$ [ $\frac{J}{\text{kg K}}$ ]	$k^2$ [ $\frac{\text{m}^2}{\text{s}}$ ]	$\Delta k^2$ [ $\frac{\text{m}^2}{\text{s}}$ ]	$\lambda$ [ $\frac{\text{N}}{\text{K s}}$ ]	$\Delta \lambda$ [ $\frac{\text{N}}{\text{K s}}$ ]
0.979	0.022	96.1	3.78E-05	0.0128	0.0003	0.00142	5E-05	0.0484	0.0020
0.682	0.027	96.1	4.08E-05	0.0199	0.0008	0.00149	7E-05	0.0788	0.0050
1.60	0.037	96.1	1.33E-04	0.0276	0.0006	0.00111	2E-05	0.0815	0.0024
1.28	0.026	96.1	1.33E-04	0.0346	0.0007	0.00102	4E-05	0.0935	0.0043
1.04	0.020	96.1	1.33E-04	0.0424	0.0008	0.00111	5E-05	0.125	0.006
0.714	0.018	96.1	1.33E-04	0.0616	0.0015	0.00098	8E-05	0.160	0.013
0.446	0.020	96.1	1.29E-04	0.0958	0.0042	0.0007	6E-05	0.178	0.017
0.867	0.023	96.1	5.12E-04	0.196	0.005	0.00059	2E-05	0.309	0.014
0.825	0.022	96.2	7.47E-04	0.301	0.008	0.00052	2E-05	0.415	0.017
0.686	0.021	96.2	0.00104	0.505	0.016	0.00043	2E-05	0.575	0.027
0.408	0.022	97	0.00103	0.843	0.046	0.00034	1E-05	0.760	0.052
0.473	0.024	97.2	0.00198	1.39	0.07	0.00037	2E-05	1.37	0.11
0.683	0.022	97.5	0.00436	2.12	0.07	0.00027	3E-05	1.50	0.17
0.586	0.027	98.1	0.00601	3.41	0.16	0.00020	1E-05	1.81	0.12
0.384	0.031	99	0.00622	5.38	0.43	0.000161	8E-06	2.30	0.22
0.297	0.024	100	0.00578	6.46	0.53	0.000123	7E-06	2.10	0.21
0.355	0.022	101.1	0.00980	9.18	0.57	8.88E-05	9.7E-06	2.16	0.27
0.583	0.031	102.3	0.0211	12.0	0.6	5.83E-05	6.9E-06	1.86	0.24
0.457	0.046	103.8	0.0202	14.7	1.5	5.57E-05	8.9E-06	2.17	0.41
0.263	0.023	106.3	0.0170	21.4	1.9	2.85E-05	2.7E-06	1.62	0.21
0.478	0.016	109.3	0.0439	30.5	1.0	1.80E-05	1.6E-06	1.46	0.14
0.369	0.023	112.8	0.0425	38.3	2.4	1.26E-05	1.4E-06	1.28	0.17
0.604	0.023	122.6	0.0791	57.1	3.0	5.44E-06	1.09E-06	0.823	0.170
0.741	0.014	130.2	0.164	73.5	1.3	3.71E-06	2.3E-07	0.723	0.046

Table II: Here  $k^2$  and finally  $\lambda$  is derived from  $t_{\max}$ .

$m$ [s]	$\Delta m$ [s]	$k$ [ $\text{ms}^{-\frac{1}{2}}$ ]	$\Delta k$ [ $\text{ms}^{-\frac{1}{2}}$ ]	$\lambda$ [ $\frac{\text{W}}{\text{mK}}$ ]	$\Delta\lambda$ [ $\frac{\text{W}}{\text{mK}}$ ]
-0.01102	5E-05	0.03334	7E-05	0.0378	0.0009
-0.00951	7E-05	0.03589	1.3E-04	0.0679	0.0027
-0.01389	9E-05	0.02969	1E-04	0.0645	0.0015
-0.01454	7E-05	0.02903	7E-05	0.0772	0.0016
-0.01460	9E-05	0.02897	9E-05	0.0942	0.0019
-0.01552	7E-05	0.02809	6E-05	0.129	0.003
-0.0155	0.0001	0.02810	1E-04	0.201	0.009
-0.0261	0.0001	0.02167	6E-05	0.327	0.009
-0.0307	0.0003	0.01998	8E-05	0.534	0.017
-0.0343	0.0001	0.01890	3E-05	0.798	0.043
-0.0361	0.0001	0.01842	3E-05	1.25	0.06
-0.04613	8E-05	0.01630	1E-05	1.49	0.05
-0.052	0.0002	0.01535	3E-05	2.13	0.10
-0.0691	0.0003	0.01331	3E-05	2.53	0.20
-0.0723	0.0005	0.01302	4E-05	2.90	0.24
-0.1513	0.0004	0.00900	1E-05	1.97	0.12
-0.1982	0.0005	0.00786	1E-05	1.97	0.10
-0.2911	0.0009	0.00649	1E-05	1.64	0.17
-0.470	0.003	0.00510	2E-05	1.48	0.13
-0.739	0.002	0.004070	6E-06	1.34	0.04
-0.972	0.002	0.003549	4E-06	1.28	0.07
-1.573	0.003	0.002790	2E-06	0.938	0.035
-2.075	0.004	0.002430	2E-06	0.893	0.048

Table III:  $m$  is the slope of the fit line shown in Figures 6 and 7 respectively. Here the thermal conductivity  $k$  is derived from  $m$ . Eventually, the heat conductivity  $\lambda$  follows from  $C'_V$  and  $k$ .

## References

- [1] Ch. Kittel, Einführung in die Festkörperphysik, 8. edition, Oldenbourg, (1989)
- [2] Instructions to F14 - Debye-Experiment, Ruprecht-Karls-Universität Heidelberg, (2005)
- [3] [http://www.crystaltechno.com/Materials/Crystal\\_Quartz.htm](http://www.crystaltechno.com/Materials/Crystal_Quartz.htm)